

Glue spin and helicity in proton from lattice QCD

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We report the first lattice QCD calculation of the glue spin S_G in the nucleon. The lattice calculation is carried out with valence overlap fermions on 2+1 flavor DWF gauge configurations on four lattice spacings and four volumes including an ensemble with physical values for the quark masses. The glue spin S_G in the $\overline{\text{MS}}$ scheme is obtained with the 1-loop perturbative matching. We find the results to be fairly insensitive to lattice spacing and quark masses. Since the frame dependence in the kinematic range $0 \leq |\vec{p}| \leq 1.5$ GeV is very mild, we take the first order large momentum expansion correction and determine the glue spin at the large momentum limit to be $S_G=0.287(55)(16)$ at the physical pion mass in the $\overline{\text{MS}}$ scheme at $\mu^2 = 10$ GeV². If the matching effect between the glue spin and helicity is neglected, this is the value for the glue helicity.

Introduction: Deep-inelastic scattering experiments reveal that contrary to the naive quark model, the quark spin contribution to the proton spin is quite small, about 25% [1, 2]. In an effort to search for the missing proton spin, recent analyses [3, 4] of the high-statistics 2009 STAR [5] and PHENIX [6] experiments at RHIC showed evidence of non-zero glue helicity Δg in the proton. For $Q^2 = 10$ GeV², the glue helicity distribution $\Delta g(x, Q^2)$ is found to be positive and away from zero in the momentum fraction region $0.05 < x$. However, the results are limited by very large uncertainty in the region $x < 0.05$.

Given the importance of $\Delta g(x)$ to explain the origin of the proton spin, and the fact that significant efforts are devoted to its precise experimental determination, a theoretical understanding and calculation of $\Delta g(x)$ is highly desired. In the infinite momentum frame (IMF), ΔG is defined as the first moment of the glue helicity distribution $\Delta g(x)$ [7],

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^+(0) | PS \rangle, \quad (1)$$

where the light front coordinates are $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$. The proton plane wave state is written as $|PS\rangle$, with momentum $P^\mu = (P, 0, 0, P)$ and polarization S . The light cone gauge-link $\mathcal{L}(\xi^-, 0) = P \exp[-ig \int_0^{\xi^-} A^+(\eta^-, 0_\perp) d\eta^-]$ is defined in the adjoint representation. It connects the gauge field tensor and its dual, $\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$, to construct a gauge invariant operator. After integrating over x , one can define

the gauge-invariant gluon helicity operator in a non-local form [8, 9],

$$\tilde{S}_g = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+} (\vec{\nabla} A^{+,b}) \mathcal{L}^{ba}(\xi^-, 0)) \right]^z \quad (2)$$

where $\nabla^+ = \partial/\partial\xi^-$. Unfortunately, one cannot evaluate this expression on the lattice directly due to its real time dependence.

On the other hand, one can access ΔG from the glue spin operator defined as

$$\vec{S}_g = \int d^3x \vec{E}^a \times \vec{A}_{\text{phys}}^a, \quad (3)$$

which comes from Chen et al.'s decomposition of the QCD angular momentum [10, 11]. In this decomposition, $A_\mu = A_{\text{phys}}^\mu + A_{\text{pure}}^\mu$. A_{pure}^μ has the same gauge transformation as A_μ and does not give rise to a field strength tensor by itself, i.e.,

$$\begin{aligned} A_{\text{pure}}^\mu &\rightarrow A_{\text{pure}}'^\mu = g(x) A_{\text{pure}}^\mu g^{-1}(x) + \frac{i}{g} g(x) \partial^\mu g^{-1}(x), \\ F_{\text{pure}}^{\mu\nu} &= \partial^\mu A_{\text{pure}}^\nu - \partial^\nu A_{\text{pure}}^\mu + ig[A_{\text{pure}}^\mu, A_{\text{pure}}^\nu] = 0. \end{aligned} \quad (4)$$

Thus $A_{\text{phys}}^\mu = A_\mu - A_{\text{pure}}^\mu$ transforms homogeneously as

$$A_{\text{phys}}^\mu \rightarrow A_{\text{phys}}'^\mu = g(x) A_{\text{phys}}^\mu g^{-1}(x). \quad (5)$$

A non-Abelian transverse condition

$$D^i A_{\text{phys}}^i = \partial^i A_{\text{phys}}^i - ig[A^i, A_{\text{phys}}^i] = 0, \quad (6)$$

is chosen for A_{phys} to have a unique solution.

A_{phys} and A_{pure} defined by Eqs. (5–6) are not Lorentz covariant and have nontrivial frame dependence [9]. It is shown in [9] that when boosted to the IMF, the solution to Eq. (6) satisfies $A_{\text{phys}}^+ = 0$, and the longitudinal component of \vec{S}_g is equivalent to the glue helicity operator \vec{S}_g .

In contrast to ΔG defined in the IMF, the matrix element of the \vec{S}_g in a finite momentum frame is calculable in lattice QCD. If one can find a gauge transformation g_c to that transforms an arbitrary gauge potential A^μ into A_c^μ ,

$$A_c^\mu = g_c^{-1} A^\mu g_c + \frac{i}{g} g_c \partial^\mu g_c^{-1}, \quad (7)$$

so that A_c^μ satisfies the condition $\vec{\partial} \cdot \vec{A}_c = 0$. Then it is easy to prove that the decomposition of A^μ given by

$$A_{\text{pure}}^\mu = \frac{i}{g} g_c \partial^\mu g_c^{-1}, \quad A_{\text{phys}}^\mu = g_c A_c^\mu g_c^{-1}.$$

can satisfy all the requirements in Eqs. (5–6). In other words, this decomposition is equivalent to the gauge invariant extension of the Coulomb gauge [12, 13].

On the lattice, such a gauge transformation g_c can be obtained numerically [14]. So the glue spin operator on the lattice can be simply defined on the Coulomb gauge fixed configuration (with the factor 2 from the normalization of the group generators),

$$\begin{aligned} \vec{S}_g &= 2 \int d^3x \text{Tr}(\vec{E} \times g_c \vec{A}_c g_c^{-1}) \\ &= 2 \int d^3x \text{Tr}(\vec{E}_c \times \vec{A}_c), \end{aligned} \quad (8)$$

where E_c and A_c are the lattice versions of the electric field and gauge potential with definitions to be addressed in the following content.

The major task of this work is calculating the matrix element of \vec{S}_g in the proton which will be indicated as S_G , in the rest and moving frames. The results are then renormalized at 1-loop order in lattice perturbation theory at $\mu^2=10 \text{ GeV}^2$, for the investigation of the frame dependence to address the matching to the helicity.

Numerical details: A preliminary attempt [15] to calculate S_G was carried out on $2+1$ flavor dynamical domain-wall configurations on a $24^3 \times 64$ lattice (24I) with the sea pion mass at 330 MeV and on a $32^3 \times 64$ lattice with sea pion mass at 300 MeV [16]. In this work, we improve the statistics on the ensembles mentioned above and carry out the calculation on another three ensembles with different lattice spacings, volumes, and sea quark masses to check the corrections to the glue spin from various systematic uncertainties. The parameters of the ensembles used in this work are listed in Table I.

The Coulomb gauge fixing condition used here is enforced by requiring that the spatial sum of the backward

TABLE I. The parameters for the RBC/UKQCD configurations [17]. $m_\pi^{(s)}$ is the pion mass of the light sea quark on the $2+1$ flavor configuration, and N_{cfg} is the number of configurations used in the simulation.

Symbol	$L^3 \times T$	$a(\text{fm})$	$m_\pi^{(s)}(\text{MeV})$	N_{cfg}
32ID	$32^3 \times 64$	0.1431(7)	170	200
48I	$48^3 \times 96$	0.1141(2)	140	81
24I	$24^3 \times 64$	0.1105(3)	330	203
32I	$32^3 \times 64$	0.0828(3)	300	309
32If	$32^3 \times 64$	0.0627(3)	370	238

difference of the HYP-smeared gauge links [18] is zero,

$$\sum_{\mu=x,y,z} [U_\mu^c(x) - U_\mu^c(x - a\hat{\mu})] = 0, \quad (9)$$

where $U_\mu^c(x)$ is the Coulomb gauge fixed Wilson link from $x + a\hat{\mu}$ to x . The gauge fixed potential A_c is defined by

$$A_{c,\mu} = \left[\frac{U_\mu^c(x) - U_\mu^{c\dagger}(x) + U_\mu^c(x - a\hat{\mu}) - U_\mu^{c\dagger}(x - a\hat{\mu})}{4ia g} \right]_{\text{traceless}} \quad (10)$$

and the color electric field used in this work is defined by the clover definition

$$\begin{aligned} F_{\mu\nu}^c &= \frac{i}{8a^2 g} (\mathcal{P}_{\mu,\nu} - \mathcal{P}_{\nu,\mu} + \mathcal{P}_{\nu,-\mu} - \mathcal{P}_{-\mu,\nu} \\ &\quad + \mathcal{P}_{-\mu,-\nu} - \mathcal{P}_{-\nu,-\mu} + \mathcal{P}_{-\nu,\mu} - \mathcal{P}_{\mu,-\nu}), \end{aligned} \quad (11)$$

where $\mathcal{P}_{\mu,\nu} = U_\mu^c(x) U_\nu^c(x + a\hat{\mu}) U_\mu^{c\dagger}(x + a\hat{\nu}) U_\nu^{c\dagger}(x)$.

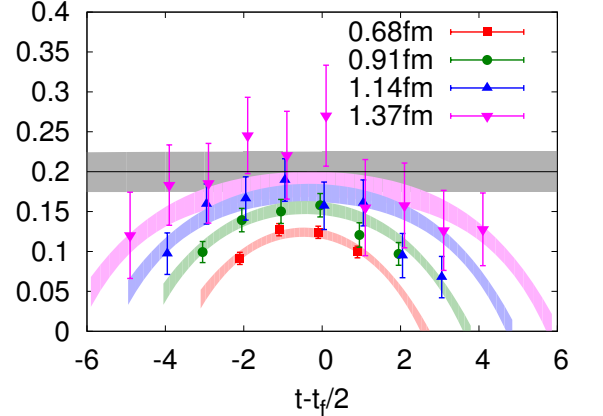


FIG. 1. The ratio $R(t_f, t)$ as a function of the source-sink separation t_f and the current time slice t , for the glue spin matrix element in the proton, S_G , is plotted at the unitary point on the 24I ensemble. The gray band shows the result extrapolated to the infinite separation, which corresponds to the prediction of S_G . The excited-state contamination is small when the source-sink separation is larger than 1 fm.

In order to extract S_G , we compute the ratio of the disconnected three-point function with the gluon operator insertion to the nucleon propagator with the source and

sink of the nucleon located at 0 and t_f , respectively. The glue spin operator is inserted at the time slice t which is between 0 and t_f . Then the ratio in a moving frame $\vec{p} = (0, 0, p_3)$ along the z -direction is

$$R(t_f, t) = \frac{\langle 0 | \Gamma_3^m \int d^3y e^{-ip_3 y_3} \chi(\vec{y}, t_f) S_g^3(t) \bar{\chi}(\vec{0}, 0) | 0 \rangle}{\langle 0 | \Gamma^e \int d^3y e^{-ip_3 y_3} \chi(\vec{y}, t_f) \bar{\chi}(\vec{0}, 0) | 0 \rangle} \quad (12)$$

where χ is the nucleon interpolation field and Γ^e and Γ_3^m are the unpolarized projection operator of the proton and the polarized one along the z -direction, respectively. When t_f is large enough, $R(t_f, t)$ is equal to the matrix element of the longitudinal glue spin operator in the proton, S_G , plus t -dependent corrections,

$$R(t_f, t) = S_G + C_1 e^{-\Delta E(t_f - t)} + C_2 e^{-\Delta E t} + C_3 e^{-\Delta E t_f}, \quad (13)$$

where ΔE is the energy difference between the first excited state and the ground state and $C_{1,2,3}$ are the spectral weights involving the excited states.

We plot the ratio $R(t_f, t)$ for the unitary point on the 24I ensemble, as a function of $t - t_f/2$ for several t_f in Fig. 1. The curves predicted by the fit agree with the data well as $\chi^2/\text{d.o.f}$ is smaller than 1.4 for all the other quark masses on five ensembles. From the fit, we see that the excited-state contamination is small when the source-sink separation is larger than 1 fm. The final prediction of S_G (the gray band) is consistent with the blue and purple data points at $t \sim t_f/2$.

It has been observed that the central values of the glue spin matrix elements as a function of HYP smearing steps are unchanged after two or three steps of smearing, as shown in Fig. 2 (for the case of the unitary point on the ensemble 24I), while the signal to noise ratio (SNR) can be improved when more HYP smearing steps are applied. In this work, 5 steps of HYP smearing are used for the glue spin operator on each ensemble, and the nucleon two-point correlators with the source located on all the time slices are generated to increase SNR. Since the tadpole improved factor is $1/u_0^5 \sim 2$ for the S_g operator without any HYP smearing, the enlargement of the result after the HYP smearing is understandable. Note that the HYP smearing here just affects the glue spin operator but the gauge action is unchanged since no reweighting is applied on the configurations.

Results: The renormalized 1-loop matrix element S_G including mixing from the quark spin is [19]

$$\begin{aligned} S_{G,(1)}^{\overline{\text{MS}}} = & \left\{ 1 - \frac{g^2}{16\pi^2} \left[N_f \left(\frac{2}{3} \log(\mu^2 a^2) - 2.41 \right) \right. \right. \\ & \left. \left. - C_A \left(\frac{4}{3} \log(\mu^2 a^2) + f_{gg}(g^2) \right) \right] \right\} S_{G,(1)}^L \\ & + \frac{g^2 C_F}{16\pi^2} \left(\frac{5}{3} \log(\mu^2 a^2) + 6.99 \right) \Delta \Sigma_{(1)}^L \\ & + O(g^4), \end{aligned} \quad (14)$$

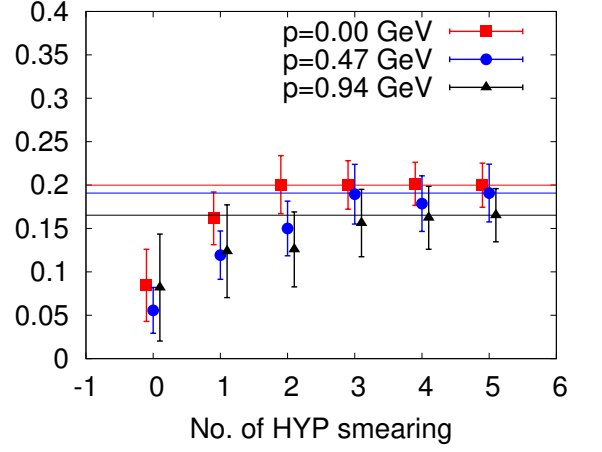


FIG. 2. The dependence of the glue spin S_G on the number of HYP smearing steps at the unitary point on the ensemble 24I, for $p = 0$ (red squares), $p = 0.47$ GeV (blue dots) and $p = 0.94$ GeV (black triangles). The values of S_G are unchanged after two or three steps of smearing.

where the superscript $\overline{\text{MS}}$ and LR indicate the quantities under the $\overline{\text{MS}}$ scheme and that under lattice regularization. We applied the Cactus improvement [20] to re-sum the major tadpole contributions to get a better convergence in the 1-loop renormalization of the glue spin, and then the finite piece $f_{gg}(g^2) \sim 1.7\text{--}2.4$ depends on the bare coupling g weakly. The details are addressed in Ref. [19]. The quark spin $\Delta \Sigma_{(1)}^L$ does not have a renormalization effect at the 1-loop level and can be replaced by the experimental value $\Delta \Sigma^{\overline{\text{MS}}}$, which is $\sim 25\%$ of the total proton spin from the global analysis of deep inelastic scattering data [1, 2].

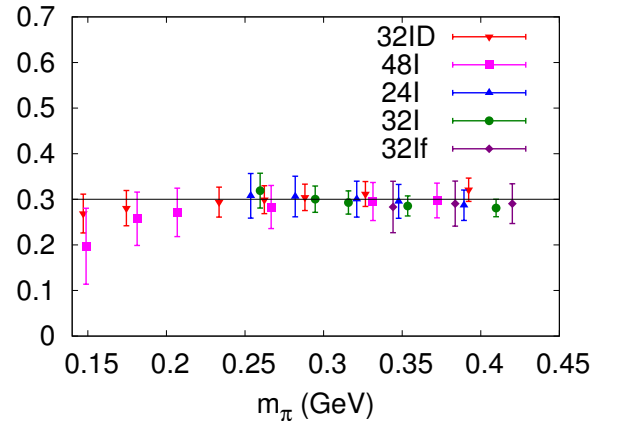


FIG. 3. The valence pion mass dependence of S_G at $\mu^2 = 10$ GeV^2 , in the rest frame. These dependencies are fairly mild and can be well described with a linear fit. A horizon line is placed at 0.3 to guide the eyes.

After the evolution to $\mu^2 = 10$ GeV^2 , we find the valence quark mass dependence is mild regardless of the

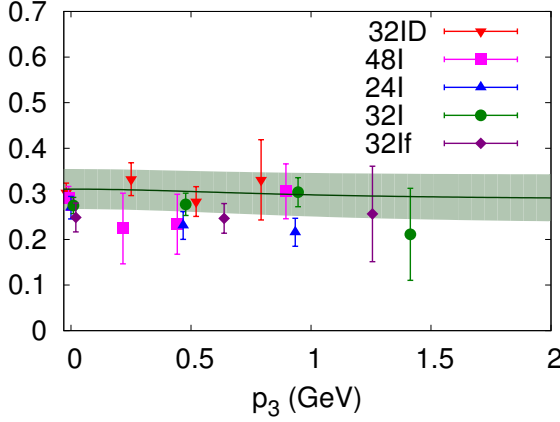


FIG. 4. The results extrapolated to the physical pion mass as a function of $|\vec{p}| = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\text{MS}}$ at 10 GeV^2 . The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit of the results.

proton momentum. In Fig. 3, we show results in the rest frame for various valence quark masses on all the five ensembles, with five pairs of volumes and lattice spacings. Both these two dependencies are also mild. To obtain S_G in a relatively large momentum frame, we calculate S_G for all the momenta smaller than $\frac{\pi}{4a}$ on all the five ensembles. To show the frame dependence, we extrapolate S_G on all the ensembles in different momentum frames to the physical value of the valence pion mass, as shown in Fig. 4. Some points are not shown in the figure if their uncertainties are larger than the signal.

The glue helicity in the proton, ΔG , corresponds to the glue longitudinal spin component S_G in the IMF. The large-momentum effective field theory (LaEMT)[21], shows a large frame dependence and finite correction at the 1-loop level,

$$\begin{aligned} \Delta G(\mu) = & \left[1 - \frac{g^2 C_A}{16\pi^2} \left(\frac{7}{3} \log \frac{(\vec{p})^2}{\mu^2} - 10.2098 \right) \right] S_G(|\vec{p}|, \mu) \\ & - \frac{g^2 C_F}{16\pi^2} \left(\frac{4}{3} \log \frac{(\vec{p})^2}{\mu^2} - 5.2627 \right) \Delta \Sigma(\mu) \\ & + O(g^4) + O\left(\frac{1}{(\vec{p})^2}\right). \end{aligned} \quad (15)$$

At $\mu^2 = 10 \text{ GeV}^2$, the finite constant term contributes at the order of 1. This indicates a convergence problem for the perturbative series even after one re-sums the large logarithms. On the other hand, the largest momentum we can have on the lattice is comparable to the proton mass, so the power corrections in Eq. (15) cannot be neglected and one cannot simply apply this matching condition. Nevertheless, the mild dependence of S_G on the proton momentum as in Fig. 4 leads us to speculate that it would be a small effect to match to the IMF, i.e., $\Delta G \approx S_G$.

So we neglect the above matching and use the following empirical form to fit our data,

$$\begin{aligned} S_G(|\vec{p}|) = & S_G(\infty) + \frac{C_1}{M^2 + |\vec{p}|^2} + C_2(m_{\pi, vv}^2 - m_{\pi, phys}^2) \\ & + C_3(m_{\pi, ss}^2 - m_{\pi, phys}^2) + C_4 a^2, \end{aligned} \quad (16)$$

where $m_{\pi, phys} = 0.139 \text{ GeV}$ and $M = 0.939 \text{ GeV}$ are the physical pion and proton mass respectively and $m_{\pi, vv/ss}$ are the valence and sea pion masses respectively. Since all the coefficients other than $S_G(\infty)$ are small, the cross term and the higher-order terms are ignored. The overall $\chi^2/\text{d.o.f.}$ is 1.21 with 110 degrees of freedom. In Fig. 4, the band of the global fit shows that the frame dependence is mild and the central value is changed by less than 10% from its value in the rest frame to that at $|\vec{p}| \sim 1.5 \text{ GeV}$; the change is smaller than the statistical uncertainty.

Since the Coulomb gauge fixing on the lattice has a build-in $O(a)$ correction, we repeated the fit with a linear term in a . The central value is changed by about 1%, while the uncertainty is larger. We take the variance of the central values from two fits as an estimate of this uncertainty. Similarly, the uncertainty from the volume dependencies $e^{-m_{\pi, vv} L}$ is estimated in the same way and added to the systematic uncertainties in quadrature. In addition, the value of the quark spin is varied by 40% to estimate the systematic uncertainty from that. The final result is $S_G(\infty, \mu^2 = 10 \text{ GeV}^2) = 0.287(55)(16)$ with two errors from the statistic and systematic uncertainties.

Summary: In this work, we calculated the glue spin in the proton for the first time based on the definition of the spin decomposition by Chen et al. [10, 11], with various quark masses, lattice spacings, volumes, and proton momenta. The results show mild dependencies on these quantities, and also a mild frame dependence. After 1-loop perturbative matching from the lattice theory to the continuum and neglecting the matching effect between the glue spin and helicity, we conclude that the gluon helicity $\Delta G(\mu^2 = 10 \text{ GeV}^2) \approx S_G(\infty, \mu^2 = 10 \text{ GeV}^2) = 0.287(55)(16)$, which is 58(11)(3)% of the total proton spin. The large values of the matching coefficients in lattice perturbation theory indicate that uncertainties can be considerable from higher-order corrections, and thus a non-perturbative renormalization should be investigated in the future.

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